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ZERO IN INDIAN MATHEMATICS: A HISTORICAL AND PHILOSOPHICAL PERSPECTIVE

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ABSTRACT

Zero, the magical number, is an indispensable part of the decimal number system. It has been recognized that one of ancient India's greatest fundamental contributions to the progress of civilization is the invention of the number zero. Playing a fundamental role in the existence of the entire domain of mathematics, it paved a way for different branches of mathematics such as algebra, analysis, calculus, and the binary code, which powers all computers. In this paper, we briefly explore the origin of the number zero, its symbolic representation, and its integration into the decimal number system. Further insight into zero and its spiritual significance in ancient civilizations is also explored. Zero represents a triumph of learning, the act of letting go of ego, and the attainment of freedom from limitations in the material world. The key focus of this paper is the journey from the invention of zero to its integration into the mystical world.

Keywords: Zero, Indian Mathematics, Brahmagupta, Bakhshali Manuscript, Decimal System, Shunya, Brahman.

I. INTRODUCTION

The significant contribution of ancient Indian Mathematics to global mathematical progress lies in the groundbreaking invention of 'zero.' This innovation proved to be a pivotal development in the advancement of civilization. In earlier mathematical systems lacking the concept of zero, alternative symbols were employed, leading to inherent ambiguities. While the idea of 'nothing' has always been understood, the conceptualization of 'nothing' as a distinct mathematical entity, both symbolically and ideologically, took considerable time to evolve. In the 5th century AD, the Indians not only embraced zero as a symbol but also fully developed its conceptual underpinnings. This achievement is attributed to the cultural significance of emptiness, nirvana, and eternity in Hinduism, providing a context that facilitated the understanding of numerical nothingness or emptiness. Zero, in isolation, may not have seemed particularly extraordinary; however, its true magic unfolded when paired with other numbers. The introduction of zero endowed Indian mathematicians with the ability to manipulate numbers on an infinite scale, a capability that set them far ahead of civilizations lacking zero in their mathematical systems. Indian astronomers, for instance, were centuries ahead of their global counterparts. India's introduction of zero into the positional number system unlocked the complete range of numerical operations, advancing maths from its early stages to a more sophisticated state, and from basic calculations to its current complexity.

In the year 1200 AD, Fibonacci, an Italian mathematician credited with introducing the decimal system to Europe, expressed the following[9]:

“The method of the Indians surpasses any known method to compute. It's a marvellous method. They do their computations using nine figures and the symbol zero”

Such advanced method of calculation, bearing striking resemblance to our contemporary approach, liberated mathematicians from laborious calculations, empowering them to address further intricate problems and explore the fundamental properties of numbers. The efficacy of the Indian technique is straightforward rules for basic mathematical operations. The ninth-century Persian mathematician, Al-Khwarizmi[9] documented the arithmetic

principles. These organized sets of instructions showcased the potential for automating certain elements of mathematics, paving the way for the eventual emergence of modern computers. Notably, the term 'algorithm,' denoting a set of precise instructions, finds its etymological roots in the name 'Al-Khwarizmi.'

The introduction of zero also revolutionized the description of fractions, providing a more accurate framework. Adding zeros to either side of a number can increase or decrease its magnitude, respectively. Introducing a decimal point and extending the number infinitely to the right corresponds to achieving precision without bounds. This level of precision proved indispensable for 17th-century mathematicians, Isaac Newton and Gottfried Leibniz[9] in their development of calculus, a field contributed to the study of continuous change.

Thus, algebra, algorithms, and calculus- the three cornerstones of modern mathematics, all trace their origins to the introduction of a symbol representing nothing.

II. VOYAGE OF ZERO

Before delving into the historical journey of zero, it is crucial to differentiate between a placeholder and a numeral. As a digit, 0 serves as a placeholder in the place value system. The concept of a placeholder involves a unique characteristic where it cannot be replicated with the same result if a particular equation is used repetitively. It's essential to note that the value of the number 0 differs from the digit 0 used in the numeral system employing positional notation. India provided fertile ground for the development of zero, transitioning from a mere placeholder to a fully-fledged numeral. The notion of place value was deeply ingrained in ancient Sanskrit terminology. Indian scholar Pingala (200 BC) used binary numbers and use the term 'shunya' to represent zero in Sanskrit. Northern India witnessed the development of numerical system and mathematical operations in Vedic period (1500-500 BC). Consequently, Indians demonstrated proficiency in handling large numbers as early as the Vedic age. Early Vedic literature contains terminology for all multiples of 10 up to 10^{18} .

1.1. The Bakhshali Manuscript: The Earliest Use of Zero

A minuscule dot on the weathered birch bark marks an earliest use of zero in the history of ancient Indian mathematics. This bark constitutes part of the revered Bakhshali manuscript, an ancient mathematical document from India, often addressed as the oldest existing manuscript in Indian mathematical history. Unearthed by a farmer in 1881 from a field in Bakhshali village near present-day Peshawar in Pakistan, this intricate document spans multiple pieces written over the course of more than a century. Radiocarbon dating, a technique measuring carbon isotopes in organic material to ascertain its age, reveals that the Bakhshali manuscript contains several sections. The oldest portion dates back to AD 224-383, followed by sections from AD 680-779 and AD 885-993. Comprising 70 leaves of birch bark, the manuscript features numerous dots, which, at the time, served not as numeric zeros, but as placeholder digits for constructing large numbers such as 101 or 1100. Merchants of that era utilized these dots for calculations. Although various ancient cultures recognized the concept of 'nothing,' they lacked a symbol or letter for it. Other civilizations, for example, the Babylonians used a double wedge, and the Mayans counted shells as placeholders. The dot representing zero didn't solely manifest in the Bakhshali manuscript; it is also apparent in an inscription on the wall of a small temple in Gwalior Fort, located in the heart of India's Madhya Pradesh, dating back to the 9th century A.D. One theory posits that the idea of zero in the form of a dot may have originated from the use of stones for calculations. When a stone was displaced, it left a round hole in its place, symbolizing the transition from something to nothing. This round hole or dent resembled a dot, possibly leading to its adoption as a placeholder. Thus, the dot denoting zero had been in use for centuries in India before its introduction to the wider world.

1.2. Aryabhatta and the Place-Value System

In the year 498 AD, the Indian mathematician, Aryabhatta[10] introduced the concept of place value with the statement 'sthānam sthānam daśa guṇam,' meaning 'From place to place, ten times in value.' The concept of 'sthāna' was explained in the mathematics chapter of the fifth-century treatise 'Āryabhaṭīya' by Āryabhaṭa, as a denominational place. Commencing with an initial invocation, the chapter outlines the initial ten notational places:

- Eka (1)
- Daśa (10)
- Śata (100)
- Sahasra (1,000)

- Ayuta (10,000)
- Niyuta (100,000)
- Prayuta (1,000,000)
- Koṭi (10,000,000)
- Arbuda (100,000,000)
- Vṛnda (1,000,000,000)

Other authors expanded this list to include more places, typically 18 or sometimes 24 [1].

Although zero was not used as a symbol by Aryabhata, French mathematician Georges Ifrah [2] proposes that the concept of zero was inherent in Aryabhata's place-value system, serving as a placeholder for powers of ten.

1.3. Brahmagupta's Formalization of Zero

The formal rules for zero emerged in Brahmagupta's work 'Brahmasphuṭa Siddhanta' ('The Opening of the Universe') [8], written in 628 AD. In this seminal text, Brahmagupta not only introduces zero but also explores negative numbers and articulates algebraic rules for basic arithmetic operations with these numbers. Some of his rules vary from the contemporary standard. Noteworthy rules of Brahmagupta include [7]:

- The sum of zero and a negative number is negative.
- The sum of zero and a positive number is positive.
- The sum of zero and zero is zero.
- The sum of a positive and a negative number is their difference, or zero if their absolute values are equal.
- A positive or negative number divided by zero results in a fraction with zero as the denominator.
- Zero divided by a negative or positive number is either zero or expressed as a fraction with zero as the numerator and a finite quantity as the denominator.
- Zero divided by zero is zero.

Brahmagupta's rule of zero divided by a zero was controversial and different from contemporary mathematical principals. However, his work laid the foundation for the systematic use of zero in arithmetic and algebra.

1.4. Bhaskara's Contributions to Zero Arithmetic

In 1152 CE, Bhaskara [3,4,5] further advanced the understanding of zero in his work Lilavati. He organized and expanded upon Brahmagupta's insights, presenting expressions such as:

- $0 + a = a$
- $0^2 = 0$
- $0^3 = 0$
- $a/0 = a/0$
- $a \cdot 0 = 0$

Bhaskara's study of zero's properties positioned him as a leading authority on zero arithmetic, and his work marked a significant step forward in the mathematical treatment of zero.

III. SPIRITUAL PERSPECTIVE OF ZERO

Ancient India, unlike many other cultures, had a philosophical predisposition toward the concept of nothingness. Practices like yoga and meditation were developed to foster mental clarity and the experience of emptiness, aligning with the teachings of Hinduism and Buddhism, which incorporate nothingness into their core philosophies. This section explores the spiritual and metaphysical dimensions of zero, linking it to Indian philosophical concepts such as Brahman, Maya, and Shunya.

1.5. Zero and Brahman: The Concept of Nothingness

Renowned Indian mythologist Devdutt Pattanaik in his TED talk narrated, "Alexander the Great once encountered an Indian gymnosophist, a sage sitting naked on a rock, gazing calmly at the sky. Curious about the man's activity, Alexander asked what he was doing. The gymnosophist said, I am experiencing nothingness. What are you doing? Alexander, proud of his achievements, responded, I am conquering the world. Both men shared a moment of laughter, each thinking the other was wasting his life in pursuit of something futile." While this tale predates the inscription of

the first zero on Gwalior's temple wall, the gymnosophist's meditation on nothingness is intricately linked to the invention of the digit zero.

In the Brihadaranyaka Upanishad [11], the phrase "**Neti Neti**" meaning "Not this, nor that" is used. It holds profound philosophical implications for existence, akin to a mathematical understanding. If we look for a numerical counterpart to the Upanishadic concept of 'Neti Neti,' we find it in zero, which is neither positive nor negative. Accordingly, the Vedas state: 'Aum Kham Brahman,' representing Brahman as shunya or zero. Thus, Brahman is equated to zero.

From a literal standpoint, zero may seem synonymous with nothingness. However, when examined from an elemental perspective, this is not the case. Without zero, the quantification of numbers such as 10s and 1,000s, would be inconceivable, reducing all figures to a singular '1.' Placing a zero before the numeral '1' would hold no significance.

Sanskrit philologist Panini[12] characterizes zero as "Adarshanam Lopah," signifying an entity that exists but remains unseen. Zero, therefore, embodies a concealed and inscrutable nature.

In the Chhandogya Upanishad[13], sage Udyalak instructs his son Shvetketu to divide a banyan tree seed into two parts. When asked about what he observes inside it, Shvetketu responds, 'I can see nothing in it.' The sage explains, 'What you perceive as emptiness is, in reality, the source of everything. From this shunya emerges a vast banyan tree.' In this context, shunya, or absolute zero, is presented as the origin of the entire Universe, symbolizing the unmanifest Brahman.

3.2. Zero and Maya: The Power of Illusion

Let's explore an alternative perspective on zero. The significance of an isolated zero remains concealed, only achieving manifestation or quantification when paired with other numerals. Similarly, Brahman exists in unmanifest state independently. It is only when combined with matter that it materializes, moving from the unmanifest state, known as nirguna, to the manifest state, saguna, with quantifiable or qualitative attributes, parallels the manner in which zero becomes evident in the presence of other integers.

Zero maintains neutrality. It doesn't assign a value to an integer; rather, it determines the value of the integer it is paired with, whether positive or negative. When combined with -1, it results in -10; with +1, it yields +10.

This analogy extends to the essence of Brahman. It is nirguna, without specific attributes, yet simultaneously represents the universal force and consciousness that influences all actions and the entirety of existence. Approaching it with a negative mindset may result in erroneous actions or destructive outcomes, whereas a positive outlook fosters constructive results.

This parallel extends to the nature of Brahman. It is nirguna, neutral, yet simultaneously the universal energy and consciousness that quantifies all actions and the entire existence. Employing it with a negative mental inclination leads to misguided actions or destruction, while a positive mindset leads to constructive outcomes.

Zero can be compared to the concept of 'Maya,' often understood as 'That which is Not.' According to Swami Swaroopananda[14], Maya cannot be definitively affirmed or denied. It is neither separate from the Divine nor identical to Reality. Maya is described as Shakti, the mysterious and incomprehensible power through which the Infinite manifests the universe. Through Maya, the singular 'One' appears as the manifold 'Many,' giving rise to various life forms and inanimate objects. The entire play of creation unfolds through Maya, and it is through Maya Shakti that the Supreme transforms into Ishwara, assuming different forms for worship. Upon removal of the Maya, every infinite form disappears, unveiling solely the supreme reality.

Likewise, 'Zero' holds the power to attribute infinite values to 'unity. Consider the power of '0' through an example involving the factorization of the expression $y^4 + 64$. Initially, finding a solution or method may seem challenging, given the expression's resemblance to $c^2 + d^2$. To address this challenge, we introduce '0' in the form of $16y^2 - 16y^2$ into the expression:

$$y^4 + 16y^2 - 16y^2 + 64$$

Simplifying, we get:

$$= (y^2 + 8)^2 - 16y^2$$

Further factorization results in:

$$= (y^2 + 8 + 4y)(y^2 + 8 - 4y)$$

The introduction of '0' has enabled us to factorize an expression that initially seemed challenging. This demonstrates the constructive influence of 'zero' in solving equations deemed difficult. The creative prowess of 'zero' becomes evident as it can be incorporated in various forms, always occurring in pairs— one positive and the other negative. Similar to Maya revealing contrasts through various aspects like positive and negative, strength and weakness, youth and age, preferences and aversions, brightness and darkness, etc., 'zero' underscores the inherent duality present in any number. The removal of Maya results in the cessation of this dual nature. Likewise, it is the existence of '0' that gives rise to negative numbers. As a consequence, each number possesses its opposite counterpart, such as 2 and -2, 3 and -3, 1/2 and -1/2.

Additionally, the combination of opposite numbers results in their collective reduction to '0':

$$2 + (-2) = 0, 3 + (-3) = 0, \text{ and so forth.}$$

However, '0' itself does not possess a sign or an opposite. There exists only one '0,' mirroring the distinctive nature of 'Maya Shakti.' Maya Shakti gives rise to opposing elements, distinctions, and a multitude of forms, it remains untouched by these distinctions. Likewise, '0' has given rise to opposing numbers, yet it lacks an opposite itself. It stands solitary in its uniqueness. Essentially, '0' holds limitless power. Much like 'Maya Shakti,' '0' can adopt infinite forms and disappear at its discretion.

3.3 Zero in Yogic Practice: The Path to Enlightenment

In yogic practices, the concept of 'shunya' can be perceived as an ever-present state of idle awareness flowing throughout the cosmos. When it intertwines with matter, it gives rise to existence or life, taking the form and essence known as consciousness. Viewing consciousness from the perspective of a space devoid of all material attributes expands our human comprehension. 'Shunya' or nothingness provides a glimpse of our origin beyond the inquisitive confines of our minds, making us conscious of our origins and the ultimate dissolution into the supreme concept of absolute awareness—a void known as 'shunya-zero.' According to the "Hatha Yoga Pradipika," the state of samadhi, the oneness of self with the cosmic Self, is inherently connected to 'shunya.' Practicing yoga and meditation guides the mind towards 'shunya' by shedding the layers of ego, desires, and attachments. Understanding 'shunya' leads to attaining enlightenment.

IV. CONCLUSION

The introduction of zero as a standalone number, stemming from the placeholder dot symbol discovered in the Bakhshali manuscript, marked a significant milestone in the history of Indian mathematics. The recognition of zero as an independent number was first documented by the astronomer and mathematician Brahmagupta in 628 CE. Beyond its mathematical significance, contemplating zero from a spiritual standpoint leads to a serene state where the burdens of emotional, physical, and mental attachments fade away- allowing individuals to connect with their true nature. Just as zero represents the absence of quantity while enabling the creation of infinite numerical values, it symbolizes the unmanifest potential of the universe, as described in Indian philosophical texts like the Upanishads.

The journey of zero—from a mere placeholder to a keystone of contemporary mathematics and a symbol of spiritual enlightenment—reflects the profound intellectual and cultural achievements of ancient India. As we continue to explore the intersections of mathematics, philosophy, and spirituality, the concept of zero remains a testament to the enduring legacy of Indian thought and its contributions to human knowledge.

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